Proof. Join AC. The three angles of the triangle ABC (fig. 14.) will not be together equal to, or greater, or less than two right angles, without the three angles of the triangle ADC being themselves also together respectively equal to, or greater, or less than two right angles, lest obviously (by the preceding) from one of those triangles be established one hypothesis, and another from the other, against the fifth, sixth, and seventh propositions of this work.

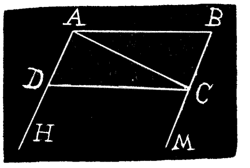

Fig. 14.

This holding good: If the four angles together of the premised quadrilateral are equal to four right angles, it follows that the three angles together of either of the just mentioned triangles will be equal to two right angles, and therefore (from the preceding) the hypothesis of right angle will be established.

But if indeed the four angles of this quadrilateral be together greater, or less than four right angles, similarly the three angles together of those triangles should be respectively either at the same time greater, or at the same time less than two right angles. Wherefore from these triangles would be established respectively (from the preceding) either the hypothesis of obtuse angle, or the hypothesis of acute angle.

Therefore by any quadrilateral, of which the four angles together are equal to, or greater, or less than four right angles, is established respectively the hypothesis of right angle, or obtuse angle, or acute angle. Quod erat demonstrandum.

COROLLARY. Hence: any two opposite sides of the premised quadrilateral being produced toward the same parts, as suppose AD to H, and BC to M; the two external angles HDC, MCD will be (Eu. I. 13.) either equal to, or less, or greater than the two internal and opposite angles together at the points A, and B, according as is true the hypothesis of right angle, or obtuse angle, or acute angle.

[To be continued.]

SOME SUGGESTIONS ABOUT VARIATION.

By ERIC DOOLITTLE, Professor of Mathematics, State University of Iowa, Iowa City, Iowa.

I think that in all of our Algebras the fundamental definitions of variation being in the idea of the ratio of two values of two changing quantities; the questions are first stated in the form of Proportions, and finally reduced to simple equations for solution.

It seems to me that this method is more cumbersome than necessary, especially in regard to the fundamental definitions, which thus involve such complex ideas that the student has a good deal of difficulty to grasp and apply

them; at least that has been my experience with young students. I have therefore given the definitions in the form of equations in the first place, which seems to considerably simplify the theorems and their application.

Assuming the student to know what variables and constants are, the presentation of the subject might be somewhat as follows.

(1). If $x, y, z, w \ldots$ are varying quantities, then v is said to be a *function* of $x, y, z, w \ldots$ when any change in the value of any or all of these variables produces a change in the value of v. Thus v will vary when $x, y, z, w \ldots$ vary.

(2). y is said to vary *as* x, if y always equals m times x, where m is constant, whatever be the value of x. This is the simplest kind of variation,* and is sometimes expressed by saying "y varies directly as x." Thus, if a train go m miles an hour, the distance (y) varies directly as the number of hours (x), since $y = mx$.

(3). y is said to vary *inversely* as x, if y always equals m times the inverse of x; that is, if $y = m(1 / x)$. Thus in the last illustration, if it require x hours for a train to go m miles, then the speed in miles per hour (y) varies inversely as x, since $y = \frac{m}{x}$.

(4). y is said to vary directly as $x, u, z, w \ldots$ if y always equals m times the product $xuzw \ldots$ where, as before, m is constant.

(5). Finally, y may be said to vary directly as certain quantities, and inversely as certain others, if y always equals m times the continued product of the former and the inverse of each of the latter. Thus y varies directly as z^2 and $a+x$, and inversely as $\sqrt[3]{x}$ if $y = mxz^2 \times (a+x) \times \frac{1}{\sqrt[3]{x}}$.

(6). The equations arising from the last four definitions may be called the *Statement* of the variation, and the first step toward the solution of any problem in variation is to write this statement. We then substitute in it such values as are known and solve for what is required. If there are several different conditions in the problem, we make the statement for each separately, and solve the resulting simultaneous equations.

Numerous examples and illustrations of these principles should of course be given, and the proof of the more elementary theorems should follow. It will be seen that they are almost self evident by this method of treating the subject.

For instance, "If y varies as x, and x varies as z, then y varies as z";

"If y varies as x, and y' varies as x', then yy' varies as xx'";

"If y varies as zx, then x varies as y / z, and z varies as y / x" etc.

"If y is a function of two variables only, x and z; and if y varies as x when z is constant but when x is constant y varies as z, then y varies with x and z at once; that is $y = mxz$."

If the above definitions be admitted, the following proof of the last

* The distinction between variation in general and the simplest possible kind of it, is here introduced to guard against the supposition that all variation is of this simplest possible kind. This danger is pointed out by Dr. Chrystal on page 275 part 1 of his Algebra.

theorem may take the place of the longer one usually given:

"Since y varies with x, multiplying x alone will multiply y by the same factor; and similarly, multiplying z alone will multiply y; hence multiplying both x and z will twice multiply y, once by each of the respective factors. Hence x and z must enter as factors of the value of y, and since there are no other variable factors, y equals a constant expression times xz. That is $y=mxz$."

I suppose this subject of variation is pretty generally omitted by Preparatory and High School classes in Algebra. It seems to me to furnish an excellent opportunity to emphasize the difference between constant and variable quantities; a distinction the student usually meets here for the first time. I think it repays a few days careful work, by the introduction it thus gives to Analytic Geometry and the Calculus. Besides, by a few obvious applications to Astronomy and Physics, it can be made of interest to the pupil.

Such an oasis, after travelling in the desert of Radicals and Imaginaries, is very welcome.

TRUE PROPOSITIONS NOT INVALIDATED BY DEFECTIVE PROOFS.

By Professor John N. Lyle, Ph. D., Westminster College, Fulton, Missouri.

A bad cause may be brilliantly advocated and a good one poorly defended. A false proposition may be supported by plausible arguments and a true one by defective and even erroneous proofs. The true proposition is not thereby shown to be false or unworthy of acceptance.

John Playfair's demonstration of the angle-sum of a rectilineal triangle may be unsatisfactory and yet the proposition that the angle-sum is two right angles may be rigorously true and its contradictory absolutely false.

Legendre's demonstration that the angle-sum can not be less than two right angles is said by Professor Halsted to be "disgraceful." Even if this be admitted, it does not follow that the proposition itself should be doubted or rejected.

Discrediting Legendre's demonstration furnishes no legitimate warrant for postulating the truth of the hypothesis that the angle-sum can be less than two right angles.

The proofs that the angle-sum can be neither greater nor less than two right angles given in the pamphlet—Euclid and the Anti-Euclidians—may fall below the standard required by rigid geometrical science, but this does not justify the acceptance as true of the assumption that the angle-sum is greater or less than two right angles.

Lobatschewsky's theorem that the angle sum can not be greater than